

Isospin-breaking two-nucleon force with explicit Δ -excitations

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We study the leading isospin-breaking contributions to the two-nucleon two-pion exchange potential due to explicit Δ degrees of freedom in chiral effective field theory. In particular, we find important contributions due to the delta mass splittings to the charge symmetry breaking potential that act opposite to the effects induced by the nucleon mass splitting.

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I. INTRODUCTION

Isospin-violating (IV) two- (2NF) and three-nucleon forces (3NF) have attracted a lot of interest in the recent years in the context of chiral effective field theory (EFT), see [1] and references therein. In the two-nucleon sector, IV one-pion [2, 3, 4, 5, 6], two-pion [4, 6, 7, 8, 9], one-pion-photon [10, 11] and two-pion-photon exchange [11, 12, 13] potentials as well as short-range contact interactions [4, 6] have been studied in this framework up to rather high orders in the EFT expansion, as also reviewed in [1]. In addition, the leading and subleading IV 3NF have been worked out in Refs. [14, 15]. As found in Ref. [6], the charge-symmetry breaking (CSB) two-pion exchange potential (TPEP) in EFT without explicit Δ degrees of freedom exhibits an unnatural convergence pattern with the (formally) subleading contribution yielding numerically the dominant effect. This situation is very similar to the isospin-conserving TPEP, where the unnaturally strong contribution at next-to-next-to-leading order in the chiral expansion can be attributed to the large values of the low-energy constants (LECs) $c_{3,4}$ accompanying the subleading $\pi\pi NN$ vertices. The large values of these LECs are well understood in terms of resonance saturation [16]. In particular, the Δ -isobar provides the dominant (significant) contribution to c_3 (c_4). Given its low excitation energy, $\Delta \equiv m_\Delta - m_N = 293$ MeV, and strong coupling to the πN system, one expects that the explicit inclusion of the Δ in EFT utilizing e.g. the so-called small scale expansion (SSE) [17]. Such a scheme allows to resum a certain class of important contributions and improves the convergence as compared to the delta-less theory. For the isospin-invariant TPEP, the improved convergence in the delta-full theory has indeed been verified via explicit calculations [18, 19]. It is natural to expect that including the Δ -isobar as an explicit degree of freedom will also improve the convergence for the IV TPEP. In our recent work [20] we already made an important step in this direction and analyzed the delta quartet mass splittings in chiral EFT. In addition, we worked out the leading Δ -contribution to IV 3NF. As expected based on resonance saturation, a significant part of the (subleading) CSB 3NF proportional to LECs $c_{3,4}$ in the delta-less theory was demonstrated to be shifted to the leading order in the delta-full theory. We also found important effects which go beyond resonance saturation of LECs $c_{3,4}$.

In this paper we derive the leading Δ -contributions to the IV two-pion exchange 2NF and compare the results with the calculations based on the delta-less theory. Our manuscript is organized as follows: in sec. II, we briefly discuss our formalism and present expressions for the IV TPEP in momentum space. We Fourier transform the potential into coordinate space and compare the obtained results at leading order in the delta-full theory with the ones found at subleading order in the delta-less theory [6] in sec. III. Our work is summarized in sec. IV.

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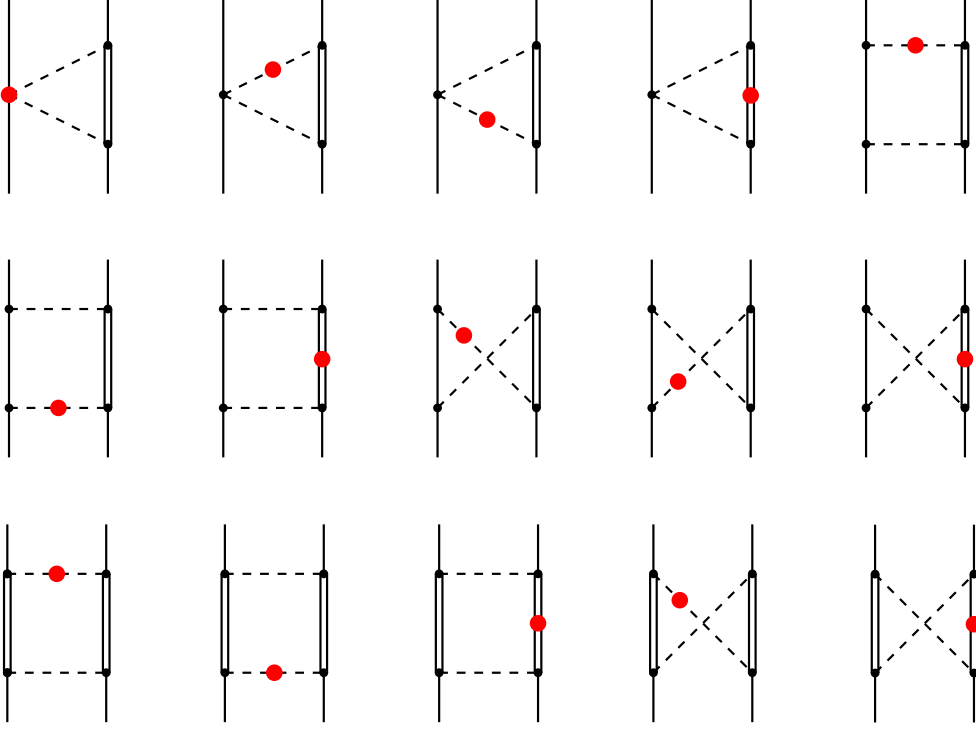


FIG. 1: Leading isospin-breaking contributions to the 2π -exchange NN potential due to intermediate Δ -excitation. Solid, dashed and double lines represent nucleons, pions and deltas, respectively. Solid dots and filled circles refer to the leading isospin-invariant and isospin-breaking vertices, in order. Diagrams resulting by the interchange of the nucleon lines are not shown.

II. Δ -CONTRIBUTIONS TO THE LEADING ISOSPIN-BREAKING TWO-PION EXCHANGE POTENTIAL

To obtain the leading isospin-violating Δ -contributions to the TPEP one has to evaluate the triangle, box and crossed-box diagrams with one insertion of isospin-breaking pion, nucleon and delta mass shifts δM_π , δm_N , δm_Δ^1 and δm_Δ^2 , respectively. In addition, one needs to consider triangle diagrams involving the leading strong IV $\pi\pi NN$ vertex. Following Ref. [1], we also include the leading electromagnetic $\pi\pi NN$ vertices. Notice that these IV $\pi\pi NN$ vertices do not contribute to the 3NF with the intermediate Δ -excitation and were not considered in [20]. We follow the same strategy as in Refs. [5, 20] and eliminate the neutron-proton mass difference term from the effective Lagrangian in favor of new isospin-violating vertices proportional to δm_N . This allows one to directly use the Feynman graph technique to derive the corresponding NN potential and thus considerably simplifies the calculations. The leading Δ -contribution to the IV TPEP arises from Feynman diagrams shown in Fig. 1. The Feynman rules for the relevant isospin-invariant vertices can be found e.g. in Ref. [21], see also [19]. To the order we are working, the Feynman rules for IV vertices after eliminating the neutron-proton mass shift have the following form.

- Two pions, no nucleons, no Δ :

$$i\delta M_\pi^2 \delta_{a3}\delta_{b3} + \delta m_N \epsilon_{3ab} (q_2 - q_1) \cdot v. \quad (2.1)$$

Here, v_μ is the baryon four-velocity, q_1 and q_2 denote the incoming pion momenta with the isospin quantum numbers a and b , respectively, $\delta M_\pi^2 \equiv M_{\pi^\pm}^2 - M_{\pi^0}^2$ is the difference of the squared charged and neutral pion masses, and $\delta m_N \equiv m_p - m_n$ is the neutron-proton mass difference. Here and in what follows, we express, whenever possible, the LECs accompanying isospin-violating vertices in terms of the pion, nucleon and delta mass shifts.

- No pions, no nucleons, one Δ :

$$i \left[(\delta m_\Delta^1 - 3\delta m_N) \frac{\tau^3}{2} \delta_{ij} + \delta m_\Delta^2 \frac{3}{4} \delta_{i3} \delta_{j3} \right] g_{\mu\nu}, \quad (2.2)$$

where i, j and μ, ν refer to the isospin and Lorentz indices of the Rarita-Schwinger field and τ^i denotes the Pauli isospin matrix with the isospin index i . Further, δm_Δ^1 and δm_Δ^2 are the two isospin-violating delta mass shifts introduced and analyzed in Ref. [20].

- Two pions, one nucleon, no Δ :

$$i \frac{\delta m_N^{\text{str}}}{2F_\pi^2} (\tau^a \delta_{b3} + \tau^b \delta_{a3} - \tau^3 \delta_{ab}) + i 2e^2 f_1 \delta_{a3} \delta_{b3}, \quad (2.3)$$

where F_π is the pion decay constant and f_1 is a LEC accompanying one of the leading-order electromagnetic operators, see Ref. [22, 23] for more details. Notice that the last term in the above expression leads to the IV 3NF, see [15], but does not generate a IV two-pion exchange 2NF at the order considered here, see also [6] for the same conclusion made using the delta-less EFT.

The leading IV NN potential in the center-of-mass system (CMS) can be conveniently written in the form:

$$V = \tau_1^3 \tau_2^3 \left[V_C^{\text{II}} + V_S^{\text{II}} \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{\text{II}} \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q} \right] + (\tau_1^3 + \tau_2^3) \left[V_C^{\text{III}} + V_S^{\text{III}} \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{\text{III}} \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q} \right], \quad (2.4)$$

where \vec{p} and \vec{p}' are the initial and final CMS momenta, $\vec{\sigma}_i$ (τ_i) refers to the spin (isospin) matrices of nucleon i and $\vec{q} \equiv \vec{p}' - \vec{p}$, $\vec{k} \equiv \frac{1}{2}(\vec{p}' + \vec{p})$. Further, the superscripts C, S, T of the scalar functions V_C, V_S and V_T denote the central, spin-spin and tensor components while the superscripts refer to the class-II and class-III isospin-violating 2NFs in the notation of Ref. [24]. The class-II interactions conserve charge symmetry and are often referred to as charge-independence breaking while the class-III 2NF are charge-symmetry breaking, see also Ref. [15]. Notice that at this order there are no class-IV contributions to the TPEP which lead to isospin-mixing. Evaluating the diagrams shown in Fig. 1 and utilizing the spectral function regularization framework [25] we obtain the following expressions for the scalar functions in Eq. (2.4):

- Δ -excitation in the triangle graphs:

$$\begin{aligned} V_C^{\text{II}} &= -\frac{h_A^2}{144F_\pi^4\pi^2\Delta\Sigma} \left(2\Delta^2\Sigma(4\Delta(3\Delta\delta m_\Delta^2 + \delta M_\pi^2) - 3\delta m_\Delta^2\Sigma) D^{\tilde{\Lambda}}(q) + 2(-M_\pi^2 + \Delta^2)(2\Delta(9\Delta\delta m_\Delta^2 + \delta M_\pi^2) \right. \\ &\quad \left. - 3\delta m_\Delta^2\omega^2) H^{\tilde{\Lambda}}(q) + 6\Sigma(8\Delta^2\delta m_\Delta^2 + \Delta\delta M_\pi^2 + \delta m_\Delta^2(\Sigma - \omega^2)) L^{\tilde{\Lambda}}(q) \right), \\ V_C^{\text{III}} &= -\frac{h_A^2\Delta}{216F_\pi^4\pi^2} (6\Delta^2(7\delta m_N - 2\delta m_N^{\text{str}} - 5\delta m_\Delta^1) + (-15\delta m_N + 6\delta m_N^{\text{str}} + 5\delta m_\Delta^1)(2\Delta^2 + \Sigma)) D^{\tilde{\Lambda}}(q) \\ &\quad -\frac{h_A^2}{216F_\pi^4\pi^2\Delta\Sigma} (-M_\pi^2 + \Delta^2)(6\Delta^2(11\delta m_N - 4\delta m_N^{\text{str}} - 5\delta m_\Delta^1) + (-15\delta m_N + 6\delta m_N^{\text{str}} + 5\delta m_\Delta^1)\omega^2) H^{\tilde{\Lambda}}(q) \\ &\quad -\frac{h_A^2}{108F_\pi^4\pi^2\Delta} (3\Delta^2(9\delta m_N - 3\delta m_N^{\text{str}} - 5\delta m_\Delta^1) + M_\pi^2(-15\delta m_N + 6\delta m_N^{\text{str}} + 5\delta m_\Delta^1)) L^{\tilde{\Lambda}}(q), \\ V_S^{\text{II}} &= V_T^{\text{II}} = V_S^{\text{III}} = V_T^{\text{III}} = 0 \end{aligned} \quad (2.5)$$

- Single Δ -excitation in the box and crossed-box graphs:

$$\begin{aligned} V_C^{\text{II}} &= \frac{g_A^2 h_A^2}{144F_\pi^4\pi^2\Delta^3\Sigma\omega^2} \left(-\Delta^2\Sigma^2(8\Delta(3\Delta\delta m_\Delta^2 + \delta M_\pi^2) + 3\delta m_\Delta^2\Sigma)\omega^2 D^{\tilde{\Lambda}}(q) \right. \\ &\quad \left. - (M_\pi^2 - \Delta^2)\omega^2(72\Delta^4\delta m_\Delta^2 + 40\Delta^3\delta M_\pi^2 - 3\delta m_\Delta^2\omega^4 - 6\Delta^2\delta m_\Delta^2(\Sigma + \omega^2) - 2\Delta\delta M_\pi^2(\Sigma + 5\omega^2)) H^{\tilde{\Lambda}}(q) \right. \\ &\quad \left. + \Sigma(2\Delta\delta M_\pi^2(2\Delta^2 + \Sigma)^2 + 2\Delta(42\Delta^3\delta m_\Delta^2 + 23\Delta^2\delta M_\pi^2 + 6\Delta\delta m_\Delta^2\Sigma + 4\delta M_\pi^2\Sigma)\omega^2 \right. \\ &\quad \left. + (-2\Delta(3\Delta\delta m_\Delta^2 + 5\delta M_\pi^2) + 3\delta m_\Delta^2\Sigma)\omega^4 - 3\delta m_\Delta^2\omega^6) L^{\tilde{\Lambda}}(q) \right), \end{aligned}$$

$$\begin{aligned}
V_S^{\text{II}} &= -q^2 V_T^{\text{II}} = \frac{g_A^2 h_A^2}{576 F_\pi^4 \pi \Delta^2} q^2 (8\Delta \delta M_\pi^2 + 3\delta m_\Delta^2 \omega^2) A^{\bar{\Lambda}}(q), \\
V_C^{\text{III}} &= -\frac{g_A^2 h_A^2}{864 F_\pi^4 \pi^2 \Delta^3 \Sigma \omega^2} \left(10\pi \Delta (3\delta m_N - \delta m_\Delta^1) \Sigma (2\Delta^2 + \Sigma)^2 \omega^2 A^{\bar{\Lambda}}(q) + 2\Delta^2 \Sigma^2 (8\Delta^2 (3\delta m_N - 5\delta m_\Delta^1) \right. \\
&\quad + 5(3\delta m_N - \delta m_\Delta^1) \Sigma) \omega^2 D^{\bar{\Lambda}}(q) + 2(-M_\pi^2 + \Delta^2) \omega^2 (120\Delta^4 (\delta m_N + \delta m_\Delta^1) + 5(3\delta m_N - \delta m_\Delta^1) \omega^4 \\
&\quad + 2\Delta^2 (3\delta m_N (\Sigma - 15\omega^2) - 5\delta m_\Delta^1 (\Sigma + \omega^2))) H^{\bar{\Lambda}}(q) + 2\Sigma (24\Delta^2 \delta m_N (2\Delta^2 + \Sigma)^2 \\
&\quad + 4\Delta^2 (\Delta^2 (33\delta m_N + 35\delta m_\Delta^1) + (9\delta m_N + 5\delta m_\Delta^1) \Sigma) \omega^2 - 5(2\Delta^2 (9\delta m_N + \delta m_\Delta^1) \\
&\quad + (3\delta m_N - \delta m_\Delta^1) \Sigma) \omega^4 + 5(3\delta m_N - \delta m_\Delta^1) \omega^6) L^{\bar{\Lambda}}(q) \Big), \\
V_S^{\text{III}} &= -q^2 V_T^{\text{III}} = \frac{g_A^2 h_A^2}{3456 F_\pi^4 \pi^2 \Delta^3 \Sigma} q^2 \left(10\pi \Delta (3\delta m_N - \delta m_\Delta^1) \Sigma \omega^2 A^{\bar{\Lambda}}(q) + 2\Delta^2 \Sigma (4\Delta^2 (3\delta m_N - 5\delta m_\Delta^1) \right. \\
&\quad + 5(3\delta m_N - \delta m_\Delta^1) \omega^2) D^{\bar{\Lambda}}(q) - 5(3\delta m_N - \delta m_\Delta^1) (2(-M_\pi^2 + \Delta^2) (4\Delta^2 - \omega^2) H^{\bar{\Lambda}}(q) - 4M_\pi^2 \Sigma L^{\bar{\Lambda}}(q)) \Big). \tag{2.6}
\end{aligned}$$

• Double Δ -excitation in the box and crossed-box graphs:

$$\begin{aligned}
V_C^{\text{II}} &= -\frac{h_A^4}{1296 F_\pi^4 \pi^2 \Delta^3 \Sigma (4\Delta^2 - \omega^2)} (-16\Delta^3 (-M_\pi^2 + \Delta^2) (3\Delta \delta m_\Delta^2 + \delta M_\pi^2) \Sigma \\
&\quad + 2\Delta^2 \Sigma (32\Delta^3 (3\Delta \delta m_\Delta^2 + \delta M_\pi^2) - 8\Delta (6\Delta \delta m_\Delta^2 + \delta M_\pi^2) \Sigma - 3\delta m_\Delta^2 \Sigma^2) \\
&\quad \times (4\Delta^2 - \omega^2) D^{\bar{\Lambda}}(q) + (8\Delta^3 (8\Delta^4 (69\Delta \delta m_\Delta^2 + 17\delta M_\pi^2) + 2\Delta^2 (63\Delta \delta m_\Delta^2 + 17\delta M_\pi^2) \Sigma + \delta M_\pi^2 \Sigma^2) \\
&\quad - 2\Delta (16\Delta^4 (87\Delta \delta m_\Delta^2 + 22\delta M_\pi^2) + 2\Delta^2 (87\Delta \delta m_\Delta^2 + 25\delta M_\pi^2) \Sigma - (6\Delta \delta m_\Delta^2 + \delta M_\pi^2) \Sigma^2) \omega^2 \\
&\quad + 4\Delta (129\Delta^3 \delta m_\Delta^2 + 37\Delta^2 \delta M_\pi^2 + 3\Delta \delta m_\Delta^2 \Sigma + 2\delta M_\pi^2 \Sigma) \omega^4 + (-2\Delta (6\Delta \delta m_\Delta^2 + 5\delta M_\pi^2) + 3\delta m_\Delta^2 \Sigma) \omega^6 \\
&\quad - 3\delta m_\Delta^2 \omega^8) H^{\bar{\Lambda}}(q) + 2\Sigma (1296\Delta^6 \delta m_\Delta^2 + 320\Delta^5 \delta M_\pi^2 + 12\Delta^3 \delta M_\pi^2 (3\Sigma - 10\omega^2) + 156\Delta^4 \delta m_\Delta^2 (\Sigma - 3\omega^2) \\
&\quad + 3\delta m_\Delta^2 \omega^4 (-\Sigma + \omega^2) - 2\Delta \delta M_\pi^2 (\Sigma^2 + 4\Sigma \omega^2 - 5\omega^4) - 6\Delta^2 \delta m_\Delta^2 (\Sigma^2 + 4\Sigma \omega^2 - 4\omega^4)) L^{\bar{\Lambda}}(q) \Big), \\
V_S^{\text{II}} &= -q^2 V_T^{\text{II}} = -\frac{h_A^4}{10368 F_\pi^4 \pi^2 \Delta^3 \Sigma} q^2 \left(2\Delta^2 \Sigma (12\Delta^2 \delta m_\Delta^2 - 8\Delta \delta M_\pi^2 - 3\delta m_\Delta^2 \omega^2) D^{\bar{\Lambda}}(q) \right. \\
&\quad + (4\Delta^2 (-4\Delta \delta M_\pi^2 + 3\delta m_\Delta^2 \Sigma) - 3\delta m_\Delta^2 (8\Delta^2 + \Sigma) \omega^2 + 3\delta m_\Delta^2 \omega^4) H^{\bar{\Lambda}}(q) - 12M_\pi^2 \delta m_\Delta^2 \Sigma L^{\bar{\Lambda}}(q) \Big), \\
V_C^{\text{III}} &= \frac{h_A^4}{3888 F_\pi^4 \pi^2 \Delta^3 \Sigma (4\Delta^2 - \omega^2)} (48\Delta^4 (-M_\pi^2 + \Delta^2) (7\delta m_N - 5\delta m_\Delta^1) \Sigma \\
&\quad - 2\Delta^2 \Sigma (96\Delta^4 (7\delta m_N - 5\delta m_\Delta^1) + 16\Delta^2 (-3\delta m_N + 5\delta m_\Delta^1) \Sigma + 5(3\delta m_N - \delta m_\Delta^1) \Sigma^2) \\
&\quad \times (4\Delta^2 - \omega^2) D^{\bar{\Lambda}}(q) + (-192\Delta^8 (89\delta m_N - 75\delta m_\Delta^1) + 5(3\delta m_N - \delta m_\Delta^1) \omega^6 (\Sigma - \omega^2) \\
&\quad - 16\Delta^6 (297\delta m_N \Sigma - 235\delta m_\Delta^1 \Sigma - 714\delta m_N \omega^2 + 590\delta m_\Delta^1 \omega^2) + 4\Delta^4 (8(-9\delta m_N + 5\delta m_\Delta^1) \Sigma^2 \\
&\quad + 5(93\delta m_N - 71\delta m_\Delta^1) \Sigma \omega^2 + 3(-229\delta m_N + 175\delta m_\Delta^1) \omega^4) + 4\Delta^2 \omega^2 (-3\delta m_N (\Sigma^2 + 19\Sigma \omega^2 - 25\omega^4) \\
&\quad + 5\delta m_\Delta^1 (\Sigma^2 + 7\Sigma \omega^2 - 9\omega^4))) H^{\bar{\Lambda}}(q) - 2\Sigma (80\Delta^6 (63\delta m_N - 53\delta m_\Delta^1) + 5(3\delta m_N - \delta m_\Delta^1) \omega^4 (\Sigma - \omega^2) \\
&\quad + 4\Delta^4 (129\delta m_N \Sigma - 115\delta m_\Delta^1 \Sigma - 495\delta m_N \omega^2 + 405\delta m_\Delta^1 \omega^2) + 2\Delta^2 (3(-7\delta m_N + 5\delta m_\Delta^1) \Sigma^2 \\
&\quad + 12(-7\delta m_N + 5\delta m_\Delta^1) \Sigma \omega^2 + 40(3\delta m_N - 2\delta m_\Delta^1) \omega^4)) L^{\bar{\Lambda}}(q) \Big), \\
V_S^{\text{III}} &= -q^2 V_T^{\text{III}} = -\frac{h_A^4}{31104 F_\pi^4 \pi^2 \Delta^3 \Sigma} q^2 \left(2\Delta^2 \Sigma (4\Delta^2 (57\delta m_N - 35\delta m_\Delta^1) + 5(3\delta m_N - \delta m_\Delta^1) \omega^2) D^{\bar{\Lambda}}(q) \right. \\
&\quad + (64\Delta^4 (-9\delta m_N + 5\delta m_\Delta^1) - 20\Delta^2 (3\delta m_N - \delta m_\Delta^1) (\Sigma - 2\omega^2) + 5(3\delta m_N - \delta m_\Delta^1) \omega^2 (\Sigma - \omega^2)) H^{\bar{\Lambda}}(q) \\
&\quad + 20M_\pi^2 (3\delta m_N - \delta m_\Delta^1) \Sigma L^{\bar{\Lambda}}(q) \Big). \tag{2.7}
\end{aligned}$$

Here, g_A , h_A and δm_N^{str} denote the nucleon, the delta-nucleon axial-vector coupling and the strong contribution to the neutron-proton mass splitting, in order. The quantities Σ , $L^{\bar{\Lambda}}$, $A^{\bar{\Lambda}}$, $D^{\bar{\Lambda}}$ and $H^{\bar{\Lambda}}$ in the above expressions are defined as follows:

$$\Sigma = 2M_\pi^2 + q^2 - 2\Delta^2,$$

$$\begin{aligned}
L^{\tilde{\Lambda}}(q) &= \theta(\tilde{\Lambda} - 2M_\pi) \frac{\omega}{2q} \ln \frac{\tilde{\Lambda}^2 \omega^2 + q^2 s^2 + 2\tilde{\Lambda} q \omega s}{4M_\pi^2(\tilde{\Lambda}^2 + q^2)}, \quad \omega = \sqrt{q^2 + 4M_\pi^2}, \quad s = \sqrt{\tilde{\Lambda}^2 - 4M_\pi^2}, \\
A^{\tilde{\Lambda}}(q) &= \theta(\tilde{\Lambda} - 2M_\pi) \frac{1}{2q} \arctan \frac{q(\tilde{\Lambda} - 2M_\pi)}{q^2 + 2\tilde{\Lambda}M_\pi}, \\
D^{\tilde{\Lambda}}(q) &= \frac{1}{\Delta} \int_{2M_\pi}^{\tilde{\Lambda}} \frac{d\mu}{\mu^2 + q^2} \arctan \frac{\sqrt{\mu^2 - 4M_\pi^2}}{2\Delta}, \\
H^{\tilde{\Lambda}}(q) &= \frac{2\Sigma}{\omega^2 - 4\Delta^2} \left[L^{\tilde{\Lambda}}(q) - L^{\tilde{\Lambda}}(2\sqrt{\Delta^2 - M_\pi^2}) \right]. \tag{2.8}
\end{aligned}$$

Notice that the spectral function cutoff $\tilde{\Lambda}$ can be set to ∞ in order to obtain the expressions corresponding to dimensional regularization. We further emphasize that the factors of Σ^{-1} in Eqs. (2.5-2.7) always show up in the combination $\Sigma^{-1}H^{\tilde{\Lambda}}(q)$ and thus do not lead to singularities.

It is instructive to verify the consistency between the results obtained in EFT with and without explicit Δ degrees of freedom as done in Ref. [19] for the isospin-conserving TPEP. Since both formulations differ from each other only by the different counting of the delta-to-nucleon mass splitting, $\Delta \sim M_\pi \ll \Lambda_\chi$ versus $M_\pi \ll \Delta \sim \Lambda_\chi$ for the delta-full and the delta-less theory, respectively. Thus expanding the various terms in Eqs. (2.5-2.7) in powers of $1/\Delta$ and counting $\Delta \sim \Lambda_\chi$ should yield either terms polynomial in momenta (i.e. contact interactions) or non-polynomial contributions absorbable into a redefinition of the LECs in the delta-less theory (in harmony with the decoupling theorem). Expanding Eqs. (2.5-2.7) in powers of $1/\Delta$ and keeping the $1/\Delta$ terms yields the following non-polynomial contributions:

$$\begin{aligned}
V_S^{\text{II}} &= -q^2 V_T^{\text{II}} = \frac{g_A^2 h_A^2 \delta M_\pi^2}{72\pi F_\pi^4 \Delta} q^2 A^{\tilde{\Lambda}}(q) + \dots, \\
V_S^{\text{III}} &= -q^2 V_T^{\text{III}} = \frac{g_A^2 h_A^2 \delta m_N}{72\pi^2 F_\pi^4 \Delta} q^2 L^{\tilde{\Lambda}}(q) + \dots, \\
V_C^{\text{III}} &= -\frac{g_A^2 h_A^2 (128M_\pi^4 + 112M_\pi^2 q^2 + 23q^4) \delta m_N L^{\tilde{\Lambda}}(q)}{216F_\pi^4 \pi^2 (4M_\pi^2 + q^2) \Delta} - \frac{h_A^2 (8M_\pi^2 + 5q^2) (2\delta m_N - \delta m^{\text{str}}) L^{\tilde{\Lambda}}(q)}{432F_\pi^4 \pi^2 \Delta} + \dots, \tag{2.9}
\end{aligned}$$

where the ellipses refer to higher-order terms. These expressions agree with the subleading IV contributions to the TPEP in the delta-less theory given in Ref. [6] if one uses the values for the LECs c_i resulting from Δ saturation:

$$c_1 = 0, \quad c_2 = -c_3 = 2c_4 = \frac{4h_A^2}{9\Delta}. \tag{2.10}$$

Notice that there is a factor $1/2$ missing in Eq. (3.52) of Ref. [6]. The correct expression for the central component of the subleading CSB TPEP in the delta-less theory $W_C^{(5)}$ reads

$$\begin{aligned}
W_C^{(5)} &= -\frac{L^{\tilde{\Lambda}}(q)}{96\pi^2 F_\pi^4} \left\{ -g_A^2 \delta m_N \frac{48M_\pi^4 (2c_1 + c_3)}{4M_\pi^2 + q^2} \right. \\
&\quad + 4M_\pi^2 \left[g_A^2 \delta m_N (18c_1 + 2c_2 - 3c_3) + \frac{1}{2} (2\delta m_N - \delta m_N^{\text{str}}) (6c_1 - c_2 - 3c_3) \right] \\
&\quad \left. + q^2 \left[g_A^2 \delta m_N (5c_2 - 18c_3) - \frac{1}{2} (2\delta m_N - \delta m_N^{\text{str}}) (c_2 + 6c_3) \right] \right\}. \tag{2.11}
\end{aligned}$$

III. RESULTS FOR THE POTENTIAL IN CONFIGURATION SPACE

We are now in the position to discuss the numerical strength of the obtained IV TPEP and to compare the results with the ones arising in the delta-less theory. The coordinate space representations of the various components of the TPEP up to NNLO are defined according to

$$\begin{aligned}
\tilde{V}(r) &= \tau_1^3 \tau_2^3 \left[\tilde{V}_C^{\text{II}}(r) + \tilde{V}_S^{\text{II}}(r) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \tilde{V}_T^{\text{II}}(r) (3\vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \right] + (\tau_1^3 + \tau_2^3) \left[\tilde{V}_C^{\text{III}}(r) + \tilde{V}_S^{\text{III}}(r) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right. \\
&\quad \left. + \tilde{V}_T^{\text{III}}(r) (3\vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \right]. \tag{3.1}
\end{aligned}$$

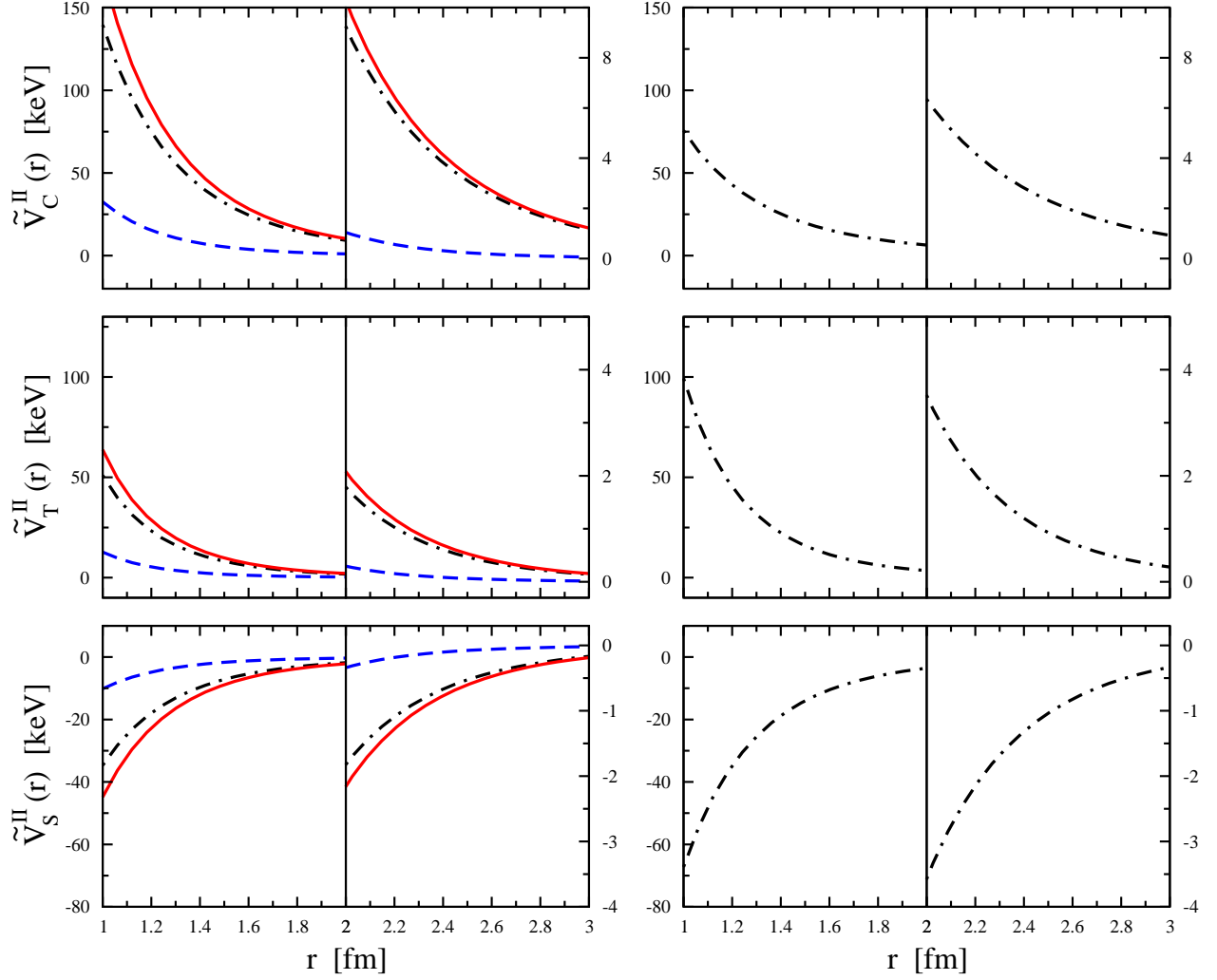


FIG. 2: Class-II two-pion exchange potential. The left (right) panel shows the results obtained at leading order in chiral EFT with explicit Δ resonances (at subleading order in chiral EFT without explicit Δ degrees of freedom). The dashed and dashed-dotted lines depict the contributions due to the delta and squared pion mass differences δm_Δ^2 and δM_π^2 , respectively, while the solid lines give the total result. In all cases, the spectral function cutoff $\tilde{\Lambda} = 700$ MeV is used.

The functions $\tilde{V}_{C,S,T}^{\text{II}}(r)$ and $\tilde{V}_{C,S,T}^{\text{III}}(r)$ can be determined for any given $r > 0$ using the spectral function representation as described in [18, 19]. We use the following values for the various LECs which appear in the TPEP: $g_A = 1.27$, $h_A = 3g_A/(2\sqrt{2}) = 1.34$ from SU(4) (or large N_c), $F_\pi = 92.4$ MeV, $\delta M_\pi^2 = 1260$ MeV² and $\delta m_N = -1.29$ MeV. For the strong nucleon mass shift, we adopt the value from Ref. [26] $\delta m_N^{\text{str}} = -2.05$ MeV, see also [27] for a recent determination from lattice QCD. The IV delta mass shifts δm_Δ^1 and δm_Δ^2 have been determined in [20] from the physical values of $m_{\Delta^{++}}$, m_{Δ^0} and either the average delta mass $\bar{m}_\Delta = 1233$ MeV leading to

$$\delta m_\Delta^1 = -5.3 \pm 2.0 \text{ MeV}, \quad \delta m_\Delta^2 = -1.7 \pm 2.7 \text{ MeV} \quad (3.2)$$

or the quark mass relation $m_{\Delta^+} - m_{\Delta^0} = m_p - m_n$ leading to

$$\delta m_\Delta^1 = -3.9 \text{ MeV}, \quad \delta m_\Delta^2 = 0.3 \pm 0.3 \text{ MeV}. \quad (3.3)$$

Let us first discuss the charge-independence-breaking contributions to the TPEP. In Fig. 2, we compare the strength of the corresponding central, spin-spin and tensor components $\tilde{V}_{C,S,T}^{\text{II}}(r)$ obtained at leading order in the delta-full theory with the ones resulting at subleading order in the EFT without explicit delta. In the former case, we add the leading-order contributions given in Eq. (3.40) of Ref. [6], see also [7] for an earlier calculation, to the leading

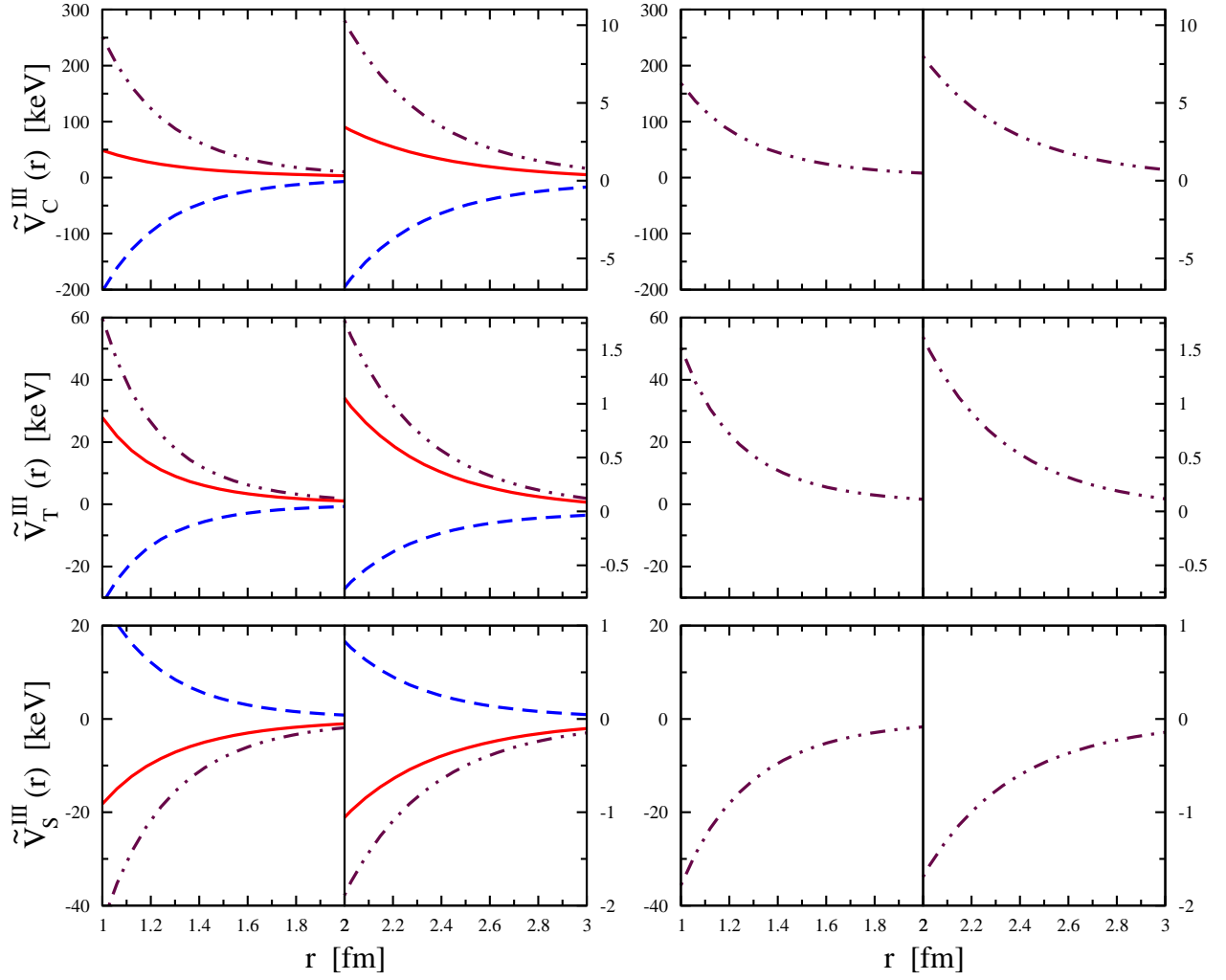


FIG. 3: Class-III two-pion exchange potential. The left (right) panel shows the results obtained at leading order in chiral EFT with explicit Δ resonances (at subleading order in chiral EFT without explicit Δ degrees of freedom and assuming charge independence of the πN coupling constant, $\beta = 0$). The dashed and dashed-double-dotted lines depict the contributions due to the delta and nucleon mass differences δm_Δ^1 and $\delta m_N^{\text{str, em}}$, respectively, while the solid lines give the total result. In all cases, the spectral function cutoff $\tilde{\Lambda} = 700$ MeV is used.

Δ -contributions in Eq. (2.5-2.7). In the latter case, we adopt the expressions given in Ref. [6] and use the central values of the LECs c_i found in Ref. [19], namely:

$$c_1 = -0.57, \quad c_2 = 2.84, \quad c_3 = -3.87, \quad c_4 = 2.89, \quad (3.4)$$

in units of GeV^{-1} . Notice that while in the delta-less theory, the leading and subleading class-II TPEP arises entirely from the pion mass difference δM_π^2 , in the delta-full theory one also finds contributions proportional to δm_Δ^2 . The results shown in Fig. 2 for the contributions $\propto \delta M_\pi^2$ are consistent with the observations made in Ref. [19] for the isospin-invariant TPEP, namely that the next-to-leading order (NLO) isovector central (spin-spin and tensor) components in the delta-full theory are overestimated (underestimated) as compared to the next-to-next-to-leading order (NNLO) calculation in the delta-less theory. We remind the reader that the charge-independence-breaking TPEP due to the pion mass difference can be expressed in terms of the corresponding isospin-invariant TPEP as demonstrated in Ref. [7]. The contributions proportional to $\propto \delta m_\Delta^2$ are numerically smaller than the ones proportional to δM_π^2 if one adopts the central value $\delta m_\Delta^2 = -1.7$ MeV and lead to a slight enhancement of the δM_π^2 -contributions. Notice that the δm_Δ^2 -terms provide a clear manifestation of effects which go beyond the subleading order in the delta-less theory, see Ref. [20] for a related discussion.

Let us now regard the charge-symmetry-breaking TPEP. Again, we compare in Fig. 3 the leading-order results in the

delta-full theory with the subleading calculations in the EFT without explicit Δ using the results of [6]. The class-III TPEP is generated in the delta-less theory by the strong and electromagnetic nucleon mass shifts and the charge dependent pion-nucleon coupling constant β [6] whose value is not known at present. In our numerical estimations, we set $\beta = 0$. In EFT with explicit Δ degrees of freedom, the class-II TPEP also receives contributions proportional to the delta mass shift δm_Δ^1 . The δm_N -parts of $V_{S,T}^{\text{III}}$ turn out to be very similar in both cases while there are sizeable deviations for V_C^{III} . Notice that although the subleading contributions in the delta-full theory have not yet been worked out and thus the convergence of the EFT expansion cannot yet be tested, the obtained results imply that the significant part of the unnaturally big subleading contribution for the class-III TPEP in the delta-less theory is now shifted to the lower order leading to a more natural convergence pattern. The improved convergence of the delta-full theory was also demonstrated for the isospin-invariant TPEP [19]. In addition to the CSB terms generated by the nucleon mass shift, there are also contributions proportional to δm_Δ^1 . For our central value, $\delta m_\Delta^1 = -5.3$ MeV, these contributions are numerically large and tend to cancel the ones proportional to δm_N and δm_N^{str} leading to a significantly weaker resulting class-III TPEP as compared to the ones at subleading order in the delta-less theory. Similar cancellations were observed recently for the IV 3NF [20]. This can be viewed as an indication that certain higher-order IV contributions still missing at subleading order in the delta-less theory are unnaturally large in the theory without explicit delta degrees of freedom. We would further like to emphasize that there is a large uncertainty in the obtained results for the IV TPEP due to the uncertainty in the values of δm_Δ^1 and δm_Δ^2 . This is visualized in Fig. 4 where the bands refer to the variation in the values $\delta m_\Delta^{1,2}$ according to Eq. (3.2).

IV. SUMMARY AND CONCLUSIONS

In this paper we have studied the leading IV contributions to the TPEP due to explicit Δ degrees of freedom. The pertinent results can be summarized as follows:

- i) We have calculated the triangle, box and crossed box NN diagrams with single and double delta excitations which give rise to the leading IV TPEP, see Fig. 1. To facilitate the calculations, we used the formulation based on the effective Lagrangian with the neutron-proton mass difference being eliminated.
- ii) We have verified the consistency of our results with the previous calculations based on the delta-less theory by expanding the non-polynomial contributions in powers of $1/\Delta$ and using resonance saturation for the LECs c_i .
- iii) We found important contributions to the IV TPEP due to the mass splittings within the delta quartet which go beyond the subleading order of the delta-less theory. In particular, the strong CSB potential found in Ref. [6] is significantly reduced by the contributions proportional to δm_Δ^1 .

In the future, it would be interesting to derive the subleading Δ -contributions to the IV TPEP in order to test the convergence of the chiral expansion in the delta-full theory. The explicit expressions for the IV TPEP worked out in this paper can (and should be) incorporated in the future partial wave analysis of nucleon-nucleon scattering.

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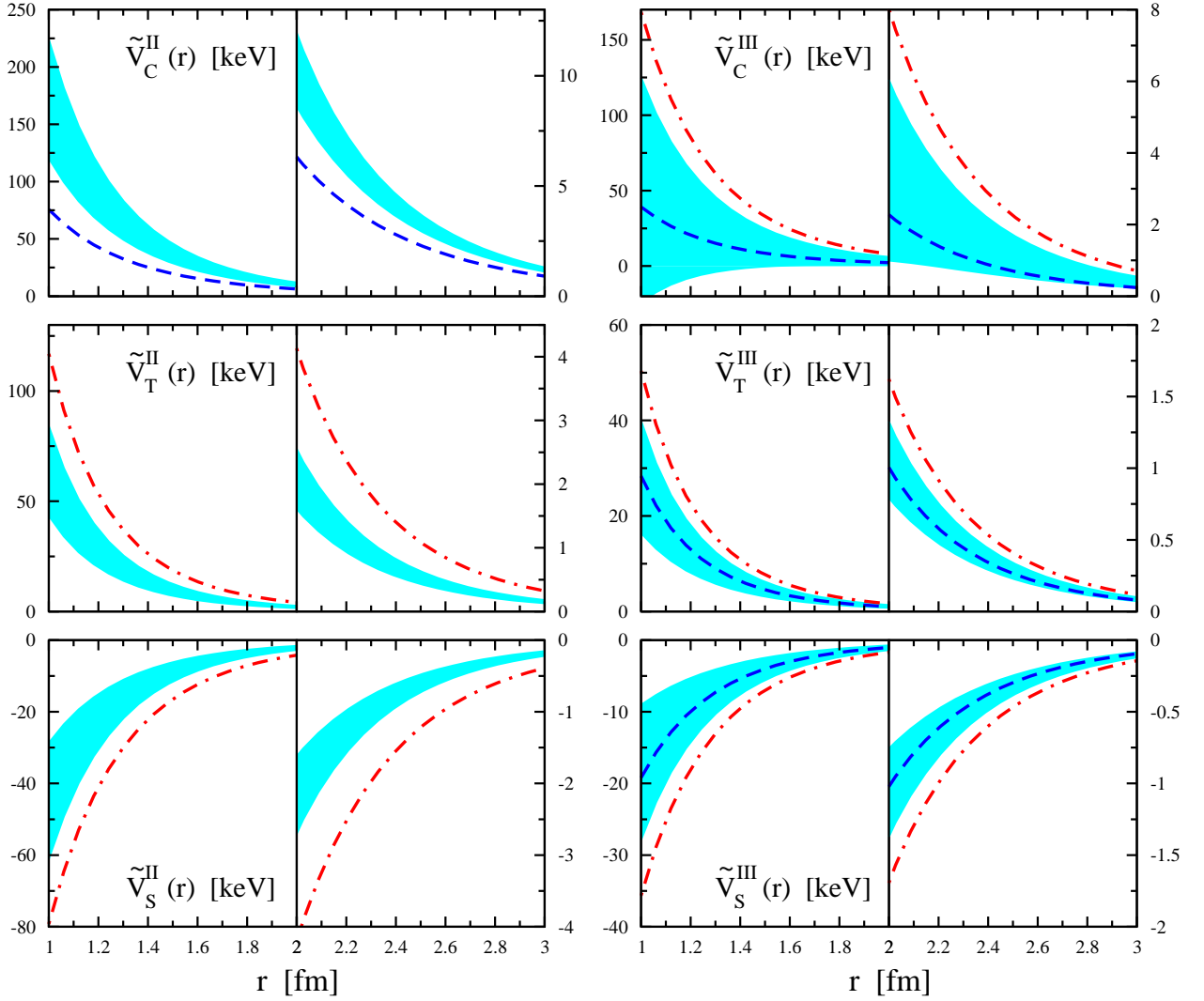


FIG. 4: Class-II and class-III two-pion exchange potentials at leading order in the delta-full theory (shaded bands) compared to the results in the delta-less theory at leading (dashed lines) and subleading (dashed-dotted lines) orders. The bands arise from the variation of δm_Δ^1 and δm_Δ^2 according to Eq. (3.2). Notice further that the leading (i.e. order- Q^4) contributions to $\tilde{V}_{T,S}^{II}(r)$ and subleading (i.e. order- Q^5) contributions to $\tilde{V}_C^{II}(r)$ vanish in the delta-less theory. In all cases, the spectral function cutoff $\tilde{\Lambda} = 700$ MeV is used.

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